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STATISTICAL IMAGE FUSION WITH GENERALISED GAUSSIAN AND ALPHA-STABLE DISTRIBUTIONS

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ABSTRACT

This paper describes a new methodology for multimodal image fusion based on non-Gaussian statistical modelling of wavelet coefficients of the input images. The use of families of generalised Gaussian and alpha-stable distributions for modelling image wavelet coefficients is investigated and methods for estimating distribution parameters are proposed. Improved techniques for image fusion are developed, by incorporating these models into the weighted average image fusion algorithm. The superior performance of the proposed methods is demonstrated using multimodal image datasets.

Index Terms— Image fusion, statistical modelling, multimodal

1. INTRODUCTION

The purpose of image fusion is to combine information from multiple images of the same scene into a single image that ideally contains all the important features from each of the original images. The resulting fused image will be thus more suitable for human and machine perception or for further image processing tasks. Many image fusion schemes have been developed in the past. As is the case with many recently proposed techniques, our developments are made using the wavelet transform, which constitutes a powerful framework for implementing image fusion algorithms [1, 2]. Specifically, methods based on multiscale decompositions consist of three main steps: first, the set of images to be fused is analysed by means of the wavelet transform, then the resulting wavelet coefficients are fused through an appropriately designed rule, and finally, the fused image is synthesized from the processed wavelet coefficients through the inverse wavelet transform. This process is depicted in Fig. 1.

The majority of early image fusion approaches, although effective, have not been based on strict mathematical foundations. Only in recent years have more rigorous approaches been proposed, including those based on estimation theory [3]. A Bayesian fusion method based on Gaussian image model has been proposed in [4]. Recent work on non-Gaussian modelling for image fusion has been proposed in [2], where the image fusion prototype method [5], combining images based on the “match and saliency” measure (variance and correlation), has been modified and applied to images modelled by symmetric α -stable distributions. In this paper we extend the work presented in [2]. We discuss different possibilities of reformulating and modifying the original Weighted Average (WA) method [5] in order to cope with more appropriate statistical model assumptions

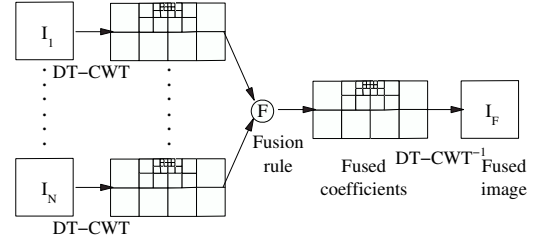


Fig. 1. Pixel-based image fusion scheme using the DT-CWT.

like the generalized Gaussian and the alpha-stable. We use a relatively novel framework, that of Mellin transform theory, in order to estimate all statistical parameters involved in the fusion algorithms.

The paper is organized as follows: In Section 2, we provide some necessary preliminaries on generalised Gaussian and alpha-stable processes and present results on the modelling of subband coefficients images. Section 3 describes the modified WA algorithms for wavelet-domain image fusion, which are based on heavy-tailed models. Section 4 compares the performance of the new algorithms with the performance of other conventional fusion techniques applied to sequences of multimodal test images. Finally, in Section 5 we conclude the paper with a short summary and suggest areas of future research.

2. STATISTICAL MODELLING OF MULTIMODAL IMAGES WAVELET COEFFICIENTS

2.1. The Generalized Gaussian Distribution

The generalized Gaussian density function proposed in [6] is given by

$$f_{s,p}(x) = \frac{1}{Z(s,p)} \cdot e^{-|x/s|^p} \quad (1)$$

where $Z(s,p) = 2\Gamma(1/p)s/p$ is a normalisation constant and $\Gamma(t) = \int_0^\infty e^{-u}u^{t-1}du$ is the well-known Gamma function

In (1), s (scale parameter) models the width of the probability density function (pdf) peak (standard deviation), while p (shape parameter) is inversely proportional to the decreasing rate of the peak. The Generalised Gaussian Distribution (GGD) model includes the Gaussian and Laplacian pdfs as special cases, corresponding to $p = 2$ and $p = 1$, respectively.

The advantage of GGD models consists in the availability of analytical expressions for their pdfs as well as of simple and efficient parameter estimators. On the other hand, Symmetric Alpha-Stable (S α S) distributions are much more flexible and rich. For example, they are also able to capture skewed characteristics.

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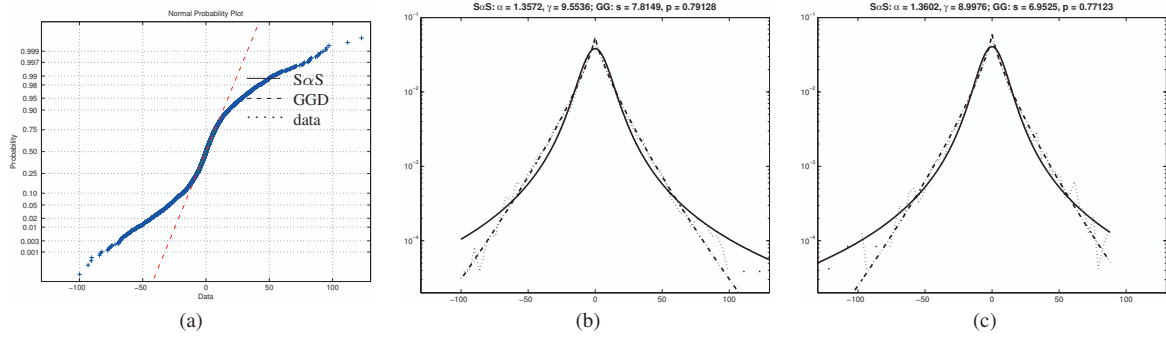


Fig. 2. An example of the corresponding normal probability plots for a hyperspectral image (band 1) (a); logarithmic probability density plots of the modelling results of hyperspectral image wavelet coefficients (1st level, 2nd orientation): (a) band 1, (b) band 2

2.2. Alpha-Stable Distributions

The S α S distribution is best defined by its characteristic function

$$\varphi(\omega) = \exp(j\delta\omega - \gamma|\omega|^\alpha), \quad (2)$$

In the equation (2) α is the *characteristic exponent*, taking values $0 < \alpha \leq 2$, δ ($-\infty < \delta < \infty$) is the *location parameter*, and γ ($\gamma > 0$) is the *dispersion* of the distribution. For values of α in the interval $(1, 2]$, the location parameter δ corresponds to the mean of the S α S distribution. The dispersion parameter γ determines the spread of the distribution around its location parameter δ , similar to the variance of the Gaussian distribution. The smaller the characteristic exponent α is, the heavier the tails of the S α S density. Gaussian processes are stable processes with $\alpha = 2$, while Cauchy processes result when $\alpha = 1$. In fact, no closed-form expressions for the general S α S pdfs are known except for these two special cases.

One consequence of heavy tails is that only moments of order less than α exist for the non-Gaussian alpha-stable family members, i.e., $E|X|^p < \infty$ for $p < \alpha$. However, Fractional Lower Order Moments (FLOM) of S α S random variables can be defined and are given by [7]:

$$E|X|^p = C(p, \alpha) \gamma^{\frac{p}{\alpha}} \quad \text{for } -1 < p < \alpha \quad (3)$$

where $C(p, \alpha) = 2^{p+1} \Gamma(\frac{p+1}{2}) \Gamma(-\frac{p}{\alpha}) / (\alpha \sqrt{\pi} \Gamma(-\frac{p}{2}))$

2.3. Modelling Results of Wavelet Subband Coefficients

The dataset used in this study, *Aviris*, contains images selected from the public AVIRIS 92AV3C hyperspectral database [8]. In this work, we analyse pairs of manually selected bands. We proceed in two steps: first, we assess whether the data deviate from the normal distribution and if they have heavy tails. To determine that, we make use of normal probability plots. An example of such a plot is shown in Fig. 2(a), demonstrating the heavy tailed characteristic of an image. Then, we check if the data is in the stable or generalized Gaussian domains of attraction by estimating the characteristic exponent α , and shape parameter p , respectively, directly from the data, with the use of the Maximum Likelihood (ML) methods. On analyzing the examples shown in Fig. 2(b)–(c) one can observe that the S α S distribution is superior to the generalized Gaussian distribution because it provides a better fit to both the mode and the tails of the empirical density of the actual data. Nevertheless, the figure demonstrates that the coefficients of different subbands and decomposition levels exhibit various degrees of non-Gaussianity. Our modelling results,

briefly shown in this section, clearly point to the need for the design of fusion rules that take into consideration the non-Gaussian heavy-tailed character of the data to achieve close to optimal image fusion performance.

3. MODEL-BASED WEIGHTED AVERAGE SCHEMES

In this section we show how the WA method can be reformulated and modified in order to cope with more appropriate statistical model assumptions like the generalized Gaussian and the alpha-stable. For the completeness of the presentation we first recall the original method (based on [5] and [2]):

1. Decompose each input image into subbands.
2. For each highpass subband pair X, Y :
 - a) Compute saliency measures, σ_x and σ_y .
 - b) Compute matching coefficient

$$M = \frac{2\sigma_{xy}}{\sigma_x^2 + \sigma_y^2}, \quad (4)$$

where σ_{xy} stands for covariance between X and Y .

- c) Calculate the fused coefficients using the formula $Z = W_x X + W_y Y$ as follows:
 - if $M > T$ ($T = 0.75$) then $W_{\min} = 0.5 \left(1 - \frac{1-M}{1-T}\right)$ and $W_{\max} = 1 - W_{\min}$ (weighted average mode, including mean mode for $M = 1$),
 - else $W_{\min} = 0$ & $W_{\max} = 1$ (selection mode),
 - if $\sigma_x > \sigma_y$ $W_x = W_{\max}$ and $W_y = W_{\min}$, else $W_x = W_{\min}$ and $W_y = W_{\max}$.
3. Average coefficients in lowpass residual.
4. Reconstruct the fused image from the processed subbands and the lowpass residual.

Essentially, the algorithm shown above considers two different modes for fusion: selection and averaging. The overall fusion rule is determined by two measures: a match measure that determines which of the two modes is to be employed and a saliency measure that determines which wavelet coefficient in the pair will be copied in the fused subband (selection mode), or which coefficient will be assigned the larger weight (weighted average mode). In the following we show how the saliency measures can be estimated adaptively, in the context of Mellin transform theory, for both GG and S α S distributions.

3.1. Saliency Estimation Using Mellin Transform

Following the arguments in [9], the use of Mellin transform has been recently proposed, as a powerful tool for deriving novel parameter estimation methods based on log-cumulants [10]. Let f be a function defined over \mathbb{R}^+ . The Mellin transform of f is defined as

$$\Phi(z) = \mathbf{M}[f(u)](z) = \int_0^{+\infty} u^{z-1} f(u) du \quad (5)$$

where z is the complex variable of the transform. By analogy with the way in which common statistics are deduced based on Fourier Transform, the following r^{th} order second-kind cumulants can be defined, based on Mellin Transform [10]

$$\tilde{k}_r = \frac{d^r \Psi(z)}{dz^r} \Big|_{z=1} \quad (6)$$

where $\Psi(z) = \log(\Phi(z))$. Following the analogy further, the method of log-moments can be applied in order to estimate the two parameters of the pdf function f (GGD or S α S). To be able to do this, the first two second-kind cumulants are required. These can be estimated empirically from N samples y_i as follows

$$\hat{k}_1 = \frac{1}{N} \sum_{i=1}^N [\log(x_i)] \quad \text{and} \quad \hat{k}_2 = \frac{1}{N} \sum_{i=1}^N [(\log(x_i) - \hat{k}_1)^2] \quad (7)$$

3.1.1. Log-moment Estimation of the GG Model

In this section we show how the saliency and match measures (4) can be computed for samples coming from GG distributions. Specifically, the variance terms appearing in the denominator of (4) need to be estimated differently, depending on which member of the GG family is considered. By plugging the expression of the GG pdf given by (1) into (5) and after some straightforward manipulations, one gets

$$\Psi(z) = \log \Phi(z) = z \log s + \log \Gamma(z) - \log p \quad (8)$$

which is the second-kind second characteristic function of a GG density. Calculating the first and second order second-kind cumulants (6) gives

$$\tilde{k}_1 = \frac{d\Psi(z)}{dz} \Big|_{z=1} = \log s + \frac{\psi_0\left(\frac{1}{p}\right)}{p} \quad (9)$$

and

$$\tilde{k}_2 = F(p) = \frac{d^2\Psi(z)}{dz^2} \Big|_{z=1} = \frac{\psi_1\left(\frac{1}{p}\right)}{p^2} \quad (10)$$

respectively, where $\psi_n(t) = \frac{d^{n+1}}{dt^{n+1}} \log \Gamma(t)$ is the polygamma function. The shape parameter p is estimated by computing the inverse of the function F . Then, p can be substituted back into the equation for \tilde{k}_1 in order to find s (and consequently the saliency measure), or a ML estimate of s could be computed from data x as

$$s = \left(\frac{p}{N} \sum_{n=1}^N |x_n|^p \right)^{1/p} = \sigma \left(\frac{\Gamma\left(\frac{1}{p}\right)}{\Gamma\left(\frac{3}{p}\right)} \right)^{1/2} \quad (11)$$

3.1.2. Log-moment Estimation of the S α S Model

For the case S α S of we obtain the following results for the second-kind cumulants of the S α S model (see [2] for detailed derivations)

$$\tilde{k}_1 = \frac{\alpha - 1}{\alpha} \psi(1) + \frac{\log \gamma}{\alpha} \quad (12)$$

and

$$\tilde{k}_2 = \frac{\pi^2 \alpha^2 + 2}{12 \alpha^2} \quad (13)$$

The estimation process simply involves now solving (13) for α and substituting back in (12) to find the value of the dispersion parameter γ (the saliency measure). We should note that this method of estimating S α S parameters was first proposed in [11]. Here, we have used an alternative derivation of the method (based on Mellin transform properties), originally proposed in [2].

Since in the case of S α S distributions classical second order moments and correlation cannot be used, new match and saliency measures need to be defined. In [2] we proposed the use of a symmetrized and normalised version of the above quantity, which enables us to define a new match measure for S α S random vectors. The symmetric covariation coefficient that we used for this purpose can be simply defined as

$$Corr_\alpha(X, Y) = \frac{[X, Y]_\alpha [Y, X]_\alpha}{[X, X]_\alpha [Y, Y]_\alpha} \quad (14)$$

where the covariation of X with Y is defined in terms of the previously introduced FLOM by [12].

$$[X, Y]_\alpha = \frac{E(XY^{<p-1>})}{E(|Y|^p)} \gamma_Y \quad (15)$$

where $x^p = |x|^p \text{sign}(x)$. It can be shown that the symmetric covariation coefficient is bounded, taking values between -1 and 1. In our implementation, the matching and similarity measures are computed locally, in a square-shaped neighbourhood of size 3×3 around each reference coefficient.

4. RESULTS

In this section, we show results obtained using the model-based approach to image fusion described in this paper. As an example, we chose to illustrate the fusion of images from AVIRIS dataset (see Section 2.3). Apart from the original WA fusion method [5], and commonly used choose-max (MAX) scheme [1], we have included the two algorithms described in this paper, i.e. the weighted average schemes based on the S α S and on GGD modelling of wavelet coefficients, and two particular cases of these, corresponding to the Cauchy (CAU) and Laplacian (LAP) densities, respectively. Two computational metrics were used to evaluate the quality of fusion: a quality index measuring similarity (in terms of illuminance, contrast and structure) between the input images and the fused images [13] (Q_1); and the Petrovic metric [14] (Q_2) measuring the amount of edge information transferred from the source images to the fused image.

The metric values, averaged over 10 pairs of dataset images are presented in Table 1. The results obtained show that the two metrics rank GGD and LAP as the best and MAX as the worst fusion method. The rankings of the remaining methods vary, however the differences between metric values are small.

The experimental results are shown in Fig. 3. Although qualitative evaluation by visual inspection is highly subjective, it seems that the best results are achieved by the GGD-based technique. In general, both modelling approaches to fusion resulted in more consistent fused images compared to slightly 'rugged' surfaces obtained with MAX. It appears that our systems, perform like feature detectors, retaining the features that are clearly distinguishable in each of the input images.

Table 1. Quality rankings of fusion methods in decreasing order

Q_1	GGD	LAP	CAU	SAS	WA	MAX
	0.928	0.928	0.928	0.927	0.926	0.920
Q_2	GGD	LAP	WA	SAS	CAU	MAX
	0.798	0.798	0.795	0.792	0.791	0.786

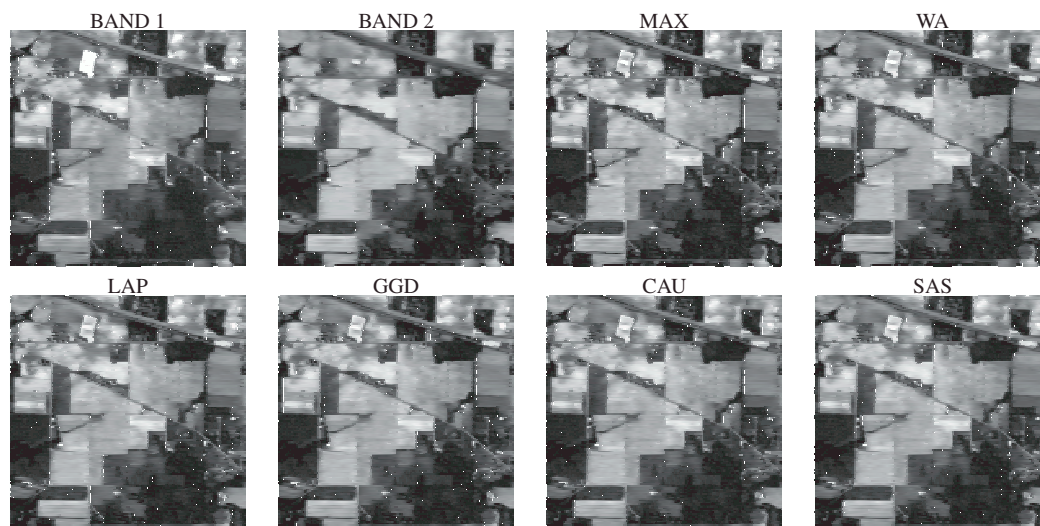


Fig. 3. Examples of the original and fused images

5. CONCLUSIONS AND FUTURE WORK

In this paper, we proposed new statistical model-based image fusion methods by reformulating the well-known WA scheme in order to account for the heavy-tailed nature of data.

We have shown through modelling experiments that images used in our experiments and their corresponding wavelet coefficients have highly non-Gaussian characteristics that can be accurately described by GGD or S α S statistical models.

In the multiscale domain, we employed the local dispersion of wavelet coefficients as saliency measure, while *symmetric covariance coefficients* were computed in order to account for the similarities between corresponding patterns in the pair of subbands to be fused. A similar approach has been applied to GGD parameters estimation, resulting in a novel estimator based on the variance of the logarithmically scaled random variable.

The fusion results show that in general the best performance is achieved by the GGD-based fusion methods followed by the S α S-based techniques.

An interesting direction in which this work could be extended is the development of algorithms that will additionally capture the inherent dependencies of wavelet coefficients across scales. This could be achieved by the use of multivariate statistical models. Research in this direction is under way and will be presented in a future communication.

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